Received: *November 13, 1997*, Accepted: *December 7, 1997*

M-fivebrane from the open supermembrane

Chong-Sun Chu

International School for Advanced Studies (SISSA/ISAS), Via Beirut 2-4, 34014 Trieste, Italy

E-mail: `cschu@sissa.it`

Ergin Sezgin*

Center for Theoretical Physics, Texas A&M University College Station, TX 77843, USA

E-mail: `sezgin@phys.tamu.edu`

Abstract: Covariant field equations of M-fivebrane in eleven dimensional curved superspace are obtained from the requirement of κ-symmetry of an open supermembrane ending on a fivebrane. The worldvolume of the latter is a (6|16) dimensional supermanifold embedded in the (11|32) dimensional target superspace. The κ-symmetry of the system imposes a constraint on this embedding, and a constraint on a modified super 3-form field strength on the fivebrane worldvolume. These constraints govern the dynamics of the M-fivebrane.

Keywords: Branes in String Theory, M- and F-theories and Other Generalizations.

*Research supported in part by NSF Grant PHY-9722090.

JHEP12(1997)001

Contents

1 Introduction

Just as open strings can end on D-branes, an eleven dimensional open supermembrane can end on M-fivebrane. This possibility was first considered in [1, 2]. Further aspects of the eleven dimensional open supermembrane were studied recently in [3, 4, 5, 6, 7]. In particular, it has been suggested in [6] that the κ-symmetry of the open supermembrane ending on a M-fivebrane may give rise to the M-fivebrane equations of motion. In this paper we will show that this is indeed the case.

In the model we consider, the worldvolume of the M-fivebrane is taken to be a super-submanifold, M, of the eleven dimensional target superspace, $\underline{M}$. The supermembrane action is an integral over a bosonic three dimensional worldvolume Σ, with its boundary $\partial\Sigma$ embedded in the supermanifold M, such that

$$\partial\Sigma \subset M \subset \underline{M}\,. \tag{1.1}$$

The requirement of κ-symmetry of the open supermembrane is shown to have the following consequences. Firstly, κ-symmetry on Σ requires that the eleven dimensional supergravity equations are satisfied. Moreover, the κ-symmetry on the boundary, which is the odd diffeomorphisms of M restricted to $\partial\Sigma$, imposes a constraint on the embedding of M in $\underline{M}$ which says that the odd tangent space of the worldsurface at any point is a subspace of the odd tangent space of the target space at the same point. Furthermore, κ-symmetry on $\partial\Sigma$ imposes a constraint on a modified super three-form field strength H defined as

$$H = dB - f^*C\,, \tag{1.2}$$

where B is the super 2-form potential on the superfivebrane worldvolume, M, and f^*C is the pullback of the target space super 3-from C to M. The superembedding constraint and the H-constraint determine completely superfivebrane equations of motion.

JHEP12(1997)001

In the superembedding approach to the description of superbranes as emphasized in [8, 9], the superembedding equation played a central role. In the case of M-fivebrane, the 3-form H was introduced for convenience in describing the field equations and it was shown that the H-constraint is a consequence of the superembedding condition [10, 11]. In the approach presented in this paper, both the superembedding condition and the H-constraint arise naturally from the requirement of κ-symmetry. The virtue of our approach becomes more apparent when we apply the formalism to super D-branes. In that case, one finds that the superembedding condition is not sufficient by itself to imply the super D-brane equations of motion, but one needs the analog of the H-constraint to do so, at least for $p \geq 6$ [12]. Thus, it is remarkable that the considerations of the κ-symmetry of the open branes ending on other branes naturally give all the constraints needed to describe the dynamics of the total system.

The nature of the supersubmanifold M may appear to be put in by hand. However, the κ-symmetry is powerful enough to restrict the nature of the possible supersubmanifolds. In particular, it is known that the $(6|16)$ dimensional submanifold is allowed [8, 9]. We use the notation $(D|D')$, where D is the real bosonic dimension and D' is the real fermionic dimension of a supermanifold. The possibility of a $(10|16)$ dimensional submanifold has been conjectured in [8], and the possibility of a $(2|16)$ dimensional submanifold has been pointed out in [6]. The determination of whether such configurations exist requires further analysis.

JHEP12(1997)001

2 Open supermembrane ending on superfivebrane

The eleven dimensional supermembrane was studied in [13, 14]. In this section, we will study an open supermembrane Σ with its boundary $\partial\Sigma$ couple to a 2-form superfield. For simplicity, we will take $\partial\Sigma$ to have a single boundary component. The membrane worldvolume is bosonic. We will take its boundary, however, to lie in a bosonic submanifold of a supermanifold M of dimension $(6|16)$, which in turn is a submanifold of a target space $\underline{M}$ of dimension $(11|32)$. We use the notations and conventions of [8]. In particular, we denote by $z^{\underline{M}} = (x^{\underline{m}}, \theta^{\underline{\mu}})$ the local coordinates on $\underline{M}$, and $A = (a, \alpha)$ is the target tangent space index. We use the ununderlined version of these indices to label the corresponding quantities on the worldsurface. The embedded submanifold M, with local coordinates y^M, is given as $z^{\underline{M}}(y)$.

We consider the following action for an open supermembrane ending on a superfivebrane

$$S = -\int_\Sigma d^3\xi \left(\sqrt{-g} + \epsilon^{ijk} C_{ijk}\right) + \int_{\partial\Sigma} d^2\sigma \epsilon^{rs} B_{rs}\,, \tag{2.1}$$

where ξ^i $(i = 0, 1, 2)$ are the coordinates on the membrane worldvolume Σ, σ^r $(r = 1, 2)$ are the coordinates on the boundary $\partial\Sigma$, g_{ij} is the metric on Σ and $g = \det g_{ij}$.

In addition to the usual super 3-form C in $(11|32)$ dimensional target superspace $\underline{M}$, we have introduced a super 2-form B on the $(6|16)$ dimensional superfivebrane worldvolume M, which is a *supersubmanifold* of $\underline{M}$. The suitable pullbacks of these superforms, and the induced metric occuring in the action are defined as:

$$
\begin{aligned}
C_{ijk} &= E_i{}^{\underline{A}} E_j{}^{\underline{B}} E_k{}^{\underline{C}} C_{\underline{CBA}}\,, \\
B_{rs} &= E_r{}^{A} E_s{}^{B} B_{BA}\,, \\
g_{ij} &= E_i{}^{\underline{a}} E_j{}^{\underline{b}} \eta_{\underline{ab}}\,,
\end{aligned}
\tag{2.2}
$$

where $\eta_{\underline{ab}}$ is the Minkowski metric in eleven dimensions, and

$$
\begin{aligned}
E_i{}^{\underline{A}} &= \partial_i z^{\underline{M}} E_{\underline{M}}{}^{\underline{A}}\,, \\
E_r{}^{A} &= \partial_r y^{M} E_{M}{}^{A}\,,
\end{aligned}
\tag{2.3}
$$

where $E_{\underline{M}}{}^{\underline{A}}$ is the target space supervielbein and $E_M{}^A$ is the worldsurface supervielbein. Defining the basis one-forms $E^{\underline{A}} = d\xi^i E_i{}^{\underline{A}}$ and $E^A = d\sigma^r E_r{}^A$, note the useful relation

$$
E^{\underline{A}}|_{\partial\Sigma} = E^A E_A{}^{\underline{A}}|_{\partial\Sigma}\,. \tag{2.4}
$$

The embedding matrix $E_A{}^{\underline{A}}$ plays an important role in the description of the model, and it is defined as

$$
E_A{}^{\underline{A}} = E_A{}^M \partial_M z^{\underline{M}} E_{\underline{M}}{}^{\underline{A}}, \tag{2.5}
$$

The action (2.1) is invariant under diffeomorphisms of Σ, with suitable boundary conditions imposed on the parameter of the transformation, as well as the tensor gauge transformations

$$
\begin{aligned}
\delta C &= d\Lambda\,, \\
\delta B &= f^*\Lambda\,,
\end{aligned}
\tag{2.6}
$$

where $\Lambda(z^{\underline{M}})$ is a super 2-form in $\underline{M}$, and the pullback $f^*\Lambda$ of a vector V on $\underline{M}$ to M is defined as

$$
(f^*V)_A = E_A{}^{\underline{A}} V_{\underline{A}}\,. \tag{2.7}
$$

We shall now require the total action to be invariant under the κ-symmetry transformation. On the interior of Σ, they take the usual form [13]

$$
\delta_\kappa z^{\underline{a}} = 0\,, \tag{2.8}
$$

$$
\delta_\kappa z^{\underline{\alpha}} = \kappa^{\underline{\gamma}}(\xi)(1+\Gamma_{(2)})_{\underline{\gamma}}{}^{\underline{\alpha}}\,, \tag{2.9}
$$

where

$$
\delta_\kappa z^{\underline{A}} = \delta_\kappa z^{\underline{M}} E_{\underline{M}}{}^{\underline{A}}\,, \tag{2.10}
$$

and

$$
\Gamma_{(2)} = \frac{1}{3!\sqrt{-g}} \epsilon^{ijk} \gamma_{ijk}\,, \tag{2.11}
$$

where the pullback γ-matrices are defined as

$$
\gamma_i = E_i{}^{\underline{a}} \Gamma_{\underline{a}}\,. \tag{2.12}
$$

JHEP12(1997)001

We also need to specify the fermionic κ-symmetry transformations of $z^{\underline{A}}$ on the boundary $\partial\Sigma$. Without loss of generality, they take the form

$$\delta_\kappa z^{\underline{a}} = 0\,, \tag{2.13}$$

$$\delta_\kappa z^{\underline{\alpha}} = \kappa^{\underline{\gamma}}(\sigma) P_{\underline{\gamma}}{}^{\underline{\alpha}} \qquad \text{on } \partial\Sigma\,, \tag{2.14}$$

where $P_{\underline{\gamma}}{}^{\underline{\alpha}}$ is some projector, whose explicit form will be spelled out later (see (2.35)).

We next derive an interesting consequence of the κ-transformations specified above. To do so, we first observe that an arbitrary transformation of y^M induces a transformation on $z^{\underline{M}}$ given by

$$\delta z^{\underline{A}} = \delta y^A E_A{}^{\underline{A}} \qquad \text{on } M, \tag{2.15}$$

where

$$\delta y^A = \delta y^M E_M{}^A\,. \tag{2.16}$$

It follows from (2.13) and (2.15) that $\delta_\kappa y^a$ and $\delta_\kappa y^\alpha$ satisfy

$$0 = \delta_\kappa y^a E_a{}^{\underline{a}} + \delta_\kappa y^\alpha E_\alpha{}^{\underline{a}}\,, \tag{2.17}$$

on the boundary $\partial\Sigma$. The $\underline{a} = b$ component of this equation is $0 = \delta_\kappa y^a E_a{}^b + \delta_\kappa y^\alpha E_\alpha{}^b$. One can check that $E_\alpha{}^b$ can be gauged away by using the bosonic diffeomorphisms of M, namely $\delta_\eta y^M E_M{}^a = \eta^a$. Hence, one can set $E_\alpha{}^b = 0$, and since $E_a{}^b$ is invertable, it follows that

$$\delta_\kappa y^a = 0 \tag{2.18}$$

on $\partial\Sigma$, and hence on M. Using this in $\underline{a} = b'$ component of (2.17), and observing that $\delta_\kappa y^\alpha$ is an arbitrary odd diffeomorphism of M, it follows that $E_\alpha{}^{b'} = 0$. Recalling that $E_\alpha{}^b = 0$ as well, we get

$$E_\alpha{}^{\underline{a}} = 0\,. \tag{2.19}$$

This is the superembedding condition that plays a crucial role in the description of superbrane dynamics [8, 9, 10].

We found the κ-transformation $\delta_\kappa y^a$ on M above. We now turn to the determination of the remaining variation $\delta_\kappa y^\alpha$ on M. Using (2.18) and (2.19) in (2.15), we find

$$\delta_\kappa y^\alpha E_\alpha{}^{\underline{\alpha}} = \delta_\kappa z^{\underline{\alpha}}\,, \tag{2.20}$$

on the boundary $\partial\Sigma$. To solve for $\delta_\kappa y^\alpha$, it is useful to introduce a normal basis $E_{A'} = E_{A'}{}^{\underline{A}} E_{\underline{A}}$ of vectors at each point on the worldsurface. The inverse of the pair $(E_A{}^{\underline{A}}, E_{A'}{}^{\underline{A}})$ is denoted by $(E_{\underline{A}}{}^A, E_{\underline{A}}{}^{A'})$ [10]. From (2.20), it follows that

$$\delta_\kappa y^\alpha = \delta_\kappa z^{\underline{\alpha}} E_{\underline{\alpha}}{}^\alpha\,, \tag{2.21}$$

on the boundary $\partial\Sigma$. This means that the variation $\delta_\kappa y^\alpha$ is an arbitrary odd-diffeomorphism, effecting the 16 fermionic coordinates of M, and that when restricted to $\partial\Sigma$, it agrees with the κ-symmetry transformation on $\underline{M}$, which also has 16 independent fermionic parameters.

JHEP12(1997)001

Now we are ready to seek the conditions for the κ-symmetry of the action (2.1).[1] Using (2.15) and (2.18) in the variation of the action, we find that the vanishing of the terms on Σ imposes constraints on the torsion super 2-form T and the super 4-form $G = dC$, such that they imply the equations of motion of the eleven dimensional supergravity [13]. The non-vanishing parts of the target space torsion are [15, 16]

$$
\begin{aligned}
T_{\underline{\alpha\beta}}{}^{\underline{c}} &= -i(\Gamma^{\underline{c}})_{\underline{\alpha\beta}}\,, \\
T_{\underline{a\beta}}{}^{\underline{\gamma}} &= -\frac{1}{36}(\Gamma^{\underline{bcd}})_{\underline{\beta}}{}^{\underline{\gamma}} G_{\underline{abcd}} - \frac{1}{288}(\Gamma_{\underline{abcde}})_{\underline{\beta}}{}^{\underline{\gamma}} G^{\underline{bcde}}\,,
\end{aligned}
\tag{2.22}
$$

and $T_{\underline{ab}}{}^{\underline{\gamma}}$. The only other non-vanishing component of G are

$$
G_{\underline{ab\gamma\delta}} = -i(\Gamma_{\underline{ab}})_{\underline{\gamma\delta}}\,. \tag{2.23}
$$

The remaining variations are on the boundary, and yield the final result

$$
\delta_\kappa S = \int_{\partial\Sigma} \epsilon^{rs} E_r^A E_s^B \delta_\kappa y^\gamma H_{\gamma BA}\,, \tag{2.24}
$$

where

$$
H = dB - f^*C\,, \tag{2.25}
$$

and satisfies the Bianchi identity[2]

$$
dH = -f^*G\,. \tag{2.26}
$$

Since $\delta_\kappa y^\alpha$ are arbitrary, the vanishing of (2.24) implies the constraint

$$
H_{\gamma BA} = 0\,. \tag{2.27}
$$

Thus the only nonvanishing component of H is H_{abc}. The constraints (2.19) and (2.27) encode elegantly all the information on the superfivebrane dynamics, as has been shown in [8, 9, 10]. For completeness, we have collected in the Appendix the covariant superfivebrane equations of motion which follow from these constraints.[3] It should be noted that the H_{abc} component does not participate in (2.27). Thus, one has the freedom to envisage a cusp-like behaviour in B_{ab} giving rise to a discontinuity in $(dB)_{abc}$. Taking into account this discontinuity, the $abcd$ component of the Bianchi identity (2.26) gets modified as[4]

$$
dH = -f^*G + \delta_W\,, \tag{2.28}
$$

[1] The κ-symmetry of the action (2.1) in a flat target superspace was also considered in [5], where the consequences of the resulting constraints are not considered. Moreover, our results for the constraints differ from theirs.

[2] The two-form B has to be rescaled by a factor of four to agree with the conventions of [10].

[3] See [17, 18] for the M-fivebrane action in the Green-Schwarz formalism, and [19] for its relation to [8, 9, 10].

[4] In [7], a modified Bianchi identity of this kind is derived by adding a bosonic piece of the M-fivebrane action to (2.1). However, the κ-symmetry of this system is by no means clear.

JHEP12(1997)001

where δ_W is the Poincaré dual of $W = \partial\Sigma$ in the bosonic fivebrane worlvolume Σ_6. A similar argument applies to the background field $C_{\underline{abc}}$, thereby modifying the $\underline{abcde}$ component of the Bianchi identity

$$dG = \delta_W \,, \tag{2.29}$$

where now δ_W is the Poincaré dual of $W = \Sigma_6$ in the eleven dimensional bosonic target space Q [20].

The modification (2.28) is important for the analysis of the reparametrization anomalies localized on $\partial\Sigma$, and (2.29) is relevant for the reparametrization anomalies on the fivebrane [21, 22]. However, these modifications are not essential for the purposes of this paper, where we derive the superfivebrane equations of motion in the bulk of its worldvolume. Hence, we shall drop the δ_W terms in the rest of this paper.

It is known that the superembedding condition (2.19) implies the H-constraint (2.27). It would be interesting to determine if the reverse is true. To this end, we have examined the the $\alpha\beta\gamma d$ component of the Bianchi identity in flat target superspace and at the linearized level in the fivebrane worldvolume fields. Interestingly, we find that the Bianchi identity (2.26) indeed implies the embedding condition at this level.

To conclude this section, we discuss the nature of the global supersymmetry of the model in a flat target superspace. Consider the transformations

$$\delta_\epsilon z^{\underline{\alpha}} = \epsilon^{\underline{\alpha}} \,, \tag{2.30}$$

where $\epsilon^{\underline{\alpha}}$ is a constant parameter. Due to (2.15), one finds that (2.30) induces a transformation on M satisfying

$$\delta_\epsilon y^\alpha E_\alpha{}^{\underline{\alpha}} = \epsilon^{\underline{\alpha}} \,. \tag{2.31}$$

It should be emphasized that these transformations, as well as the κ-symmetry transformations are not special cases of the Λ-transformations. The latter is a transformation of the background fields involving a parameter that is an arbitrary function of the target space coordinates.

Substituting the variation (2.31), we find that the action (2.1) is invariant, upon the use of the H-constraint (2.27). It remains to analyse the consequences of the condition (2.31). Multiplying (2.31) with $E_{\underline{\alpha}}{}^{\beta'}$ (defined earlier), we get

$$\epsilon^{\underline{\alpha}} E_{\underline{\alpha}}{}^{\beta'} = 0 \,. \tag{2.32}$$

The target space spinor index $\underline{\alpha}$, running fom 1 to 32 is split in two, $\underline{\alpha} \to (\alpha, \alpha')$, where both the worldsurface index α and the normal index α' run from 1 to 16.

To elucidate the meaning of this condition on the parameter $\epsilon^{\underline{\alpha}}$, it is useful to define the projection operators

$$\begin{aligned} E_{\underline{\alpha}}{}^{\alpha} E_\alpha{}^{\underline{\gamma}} &= \tfrac{1}{2}(1 + \Gamma_{(5)})_{\underline{\alpha}}{}^{\underline{\gamma}} \,, \\ E_{\underline{\alpha}}{}^{\alpha'} E_{\alpha'}{}^{\underline{\gamma}} &= \tfrac{1}{2}(1 - \Gamma_{(5)})_{\underline{\alpha}}{}^{\underline{\gamma}} \,, \end{aligned} \tag{2.33}$$

JHEP12(1997)001

where $\Gamma_{(5)}$ satisfies $\Gamma_{(5)}^2 = 1$. This matrix is defined by (2.33) and is given in the Appendix. Multiplying (2.32) with $E_{\beta'}{}^{\underline{\beta}}$ gives

$$\epsilon^{\underline{\alpha}}(1-\Gamma_{(5)})_{\underline{\alpha}}{}^{\underline{\beta}} = 0\,. \tag{2.34}$$

This means that at most half of the supersymmetry can survive. Furthermore, since the matrix $\Gamma_{(5)}$ defined in (A.3) is a complicated function of the worldvolume fields, the condition (2.34) severely restricts the allowed configurations for them. One possibility is to set all the worldvolume fields equal to zero. In that case, and in a physical gauge, $\Gamma_{(5)}$ becomes a product of constant worldvolume γ-matrices, implying that half of target space supersymmetry.

Finally, we note that just as $\epsilon^{\underline{\alpha}}$ satisfies (2.34), the variation $\delta_\kappa z^{\underline{\alpha}}$ given in (2.14) satisfies $\bar{\kappa}P(1-\Gamma_{(5)}) = 0$ on the boundary $\partial\Sigma$. This can be seen by multiplying the $\underline{A} = \underline{\alpha}$ component of (2.15) by $E_{\underline{\alpha}}{}^{\alpha'}E_{\alpha'}{}^{\underline{\gamma}}$ and noting that $E_\alpha{}^{\underline{\alpha}}E_{\underline{\alpha}}{}^{\beta'} = 0$. This condition can be satisfied by taking

$$P = \tfrac{1}{2}(1+\Gamma_{(5)}). \tag{2.35}$$

3 Boundary conditions

In this secton we consider the boundary conditions that arise from the variation of the action (2.1). The requirement of the action be stationary when the supermembrane field equations of [14] hold imposes the boundary condition

$$\int_{\partial\Sigma}(\delta y^A E_A{}^{\underline{a}}\sqrt{-g}n^i E_{i\underline{a}} + \delta y^C n_i \epsilon^{ijk}E_j{}^B E_k{}^A H_{ABC}) = 0\,. \tag{3.1}$$

Using (2.19) and (2.27), we obtain

$$\int_{\partial\Sigma}[\delta z^{a'}\sqrt{-g}n^i E_{ia'} + \delta y^c(\sqrt{-g}E_c{}^{\underline{a}}n^i E_{i\underline{a}} + n_i\epsilon^{ijk}E_j{}^b E_k{}^a H_{abc})] = 0\,, \tag{3.2}$$

where $\delta z^{a'}$ is defined in (2.10). This equation is satisfied by imposing the Dirichlet boundary condition,

$$\delta z^{a'}|_{\partial\Sigma} = 0\,, \tag{3.3}$$

and the Neumann boundary condition

$$\left(\sqrt{-g}n^i E_{ic} + n_i\epsilon^{ijk}E_j{}^a E_k{}^b H_{abc}\right)|_{\partial\Sigma} = 0\,, \tag{3.4}$$

where n^i is a unit vector normal to the boundary $\partial\Sigma$, and a' labels the directions transverse to the fivebrane worldvolume. We expect that this constraint is κ-invariant, modulo the fermionic field equation of the superfivebrane (see the Appendix) [10]

$$\mathcal{E}_a(1-\Gamma_{(5)})\gamma^b m_b{}^a = 0\,. \tag{3.5}$$

A boundary condition similar to (3.4) has also been discussed in [6] for a flat target space and a purely bosonic 2-form on the fivebrane worldvolume. These authors set the

JHEP12(1997)001

C-dependent term of H in (3.4) equal to zero separately by imposing suitable boundary condition on the fermionic variables, involving a projector of the form $(1+\Gamma_{(5)})$ with $\Gamma_{(5)}$ defined as in (A.3).

In [3], on the other hand, the 2-form field B is not considered, and the C-dependent terms are set equal to zero at the boundary, by projecting the fermionic variables in particular gauges. In this case, one has to check that the boundary terms due to the global supersymmetry variation of the supermembrane action vanish, and indeed they do [3]. It should be emphasized that this is possible only by sacrificing the eleven dimensional super Poincaré invariance [14]. For example, the boundary terms cannot be made to vanish for an open supermembrane with boundaries moving freely in $\underline{M}$.

For completeness, we also consider the boundary terms due to the reparametrization transformations

$$\delta z^{\underline{M}} = v^i \partial_i z^{\underline{M}} , \tag{3.6}$$

under which the action transforms as

$$\delta S = \int_\Sigma d^3\xi \partial_i (v^i \mathcal{L}) . \tag{3.7}$$

This boundary term vanishes by imposing the condition

$$n_i v^i |_{\partial\Sigma} = 0 . \tag{3.8}$$

Requiring that the boundary condition (3.4) is preserved by the reparametrization transformations imposes the conditions

$$n^i \partial_i v^r |_{\partial\Sigma} = 0 . \tag{3.9}$$

4 Comments

In this paper we considered an open supermembrane with a bosonic worldvolume Σ ending on a superfivebrane with worldvolume M, which is a $(6|16)$ dimensional supersubmanifold of the $(11|32)$ dimensional target superspace $\underline{M}$. We showed that the requirement of κ-symmetry not only constraints the eleven dimensional curved background fields to satisfy their equations of motion, but also yields a superembedding condition and a constraint on a modified 3-form field strength, H, which determine the superfivebrane equations of motion.

Our formalism can be applied to other possible choices of embeddings (1.1). In particular there are two special cases which deserve further analysis within the present framework. In one case, the supermembrane ends on a superstring, and thus $\partial\Sigma = M_B$, where M_B is the bosonic part of the $(2|16)$ dimensional string superworldsheet. In another case, the supermembrane ends on the boundary of $\underline{M}$, namely $M = \partial\underline{M}$, corresponding to a M-ninebrane. The latter case is similar to the configuration considered in [23]. For a κ-symmetric formulation of an open supermembrane ending on the Horawa-Witten ninebrane, see [4, 5].

JHEP12(1997)001

In a separate paper [24], we will show that the ideas presented here apply also to fundamental type II strings ending on D-branes, $D2$-brane ending on solitonic type IIA fivebrane, and open Dp_1-branes ending on Dp_2-branes.

The application of our formalism to type I open branes, i.e. branes in a target space with sixteen real supersymmetries, should also be possible. It would be interesting, for example, to derive the equations of motion for the heterotic fivebrane, via the study of heterotic string ending on solitonic fivebrane.

Another possible generalization of the present work is to consider open branes ending on branes that do not possess maximal transverse space rotational symmetry. For example, the *pp* waves and the sixbranes, also known as Kaluza-Klein monopoles, in eleven dimensions can be considered as possible end point surfaces for the eleven dimensional supermembrane.

One may also consider elevating the open brane worldvolumes considered here to supermanifolds.[5] Although an action is not known for such systems, one may nonetheless obtain the equations of motion for all the branes involved by generalizing the usual superembedding approach to deal with triplet (1.1) of supermanifolds Σ, M and $\underline{M}$.

Finally, a matrix regularization [25, 26, 27, 6] of the action (2.1) may be relevant for the generalization of M(atrix) theory [28] in a curved background [29] and in the presence of fivebrane [30].

Acknowledgments

We are very grateful to Paul Howe for helpful e-mail communications and Per Sundell for useful discussions. E.S. would like to thank the International Center for Theoretical Physics for hospitality.

A M-fivebrane equations of motion

Here, we give the nonlinear field equations of the superfivebrane equations, up to second order fermionic terms, that follow from the superembedding condition (2.19), which are proposed to arise equally well from the H-constraint (2.27). These equations are [8, 9, 10]:

$$\mathcal{E}_a(1-\Gamma_{(5)})\gamma^b m_b{}^a = 0\,, \tag{A.1}$$

$$G^{mn}\nabla_m H_{npq} = \frac{3\sqrt{-g}}{128(1-\frac{2}{3}\mathrm{tr}\,k^2)}(1-\tfrac{2}{3}k)^4{}_{[p}{}^n\ \epsilon_{q]nm_1\cdots m_4}\ G^{m_1\cdots m_4}\,,$$

$$G^{mn}\nabla_m\mathcal{E}_n{}^{\underline{c}} = \frac{(1-\frac{2}{3}\mathrm{tr}\,k^2)}{6!\sqrt{-g}}\epsilon^{m_1\cdots m_6}(G^{\underline{a}}{}_{m_1\cdots m_6} + \tfrac{2}{3}G^{\underline{a}}{}_{m_1m_2m_3}\,H_{m_4m_5m_6}\,)(\delta_{\underline{a}}{}^{\underline{c}} - \mathcal{E}_{\underline{a}}{}^m\mathcal{E}_m{}^{\underline{c}})\,,$$

where

$$\mathcal{E}_m{}^{\underline{a}}(x) \;=\; \partial_m z^{\underline{M}}E_{\underline{M}}{}^{\underline{a}} \qquad \text{at } \theta=0\,,$$

[5] We thank Paul Howe for a discussion about this possibility.

JHEP12(1997)001

$$\mathcal{E}_m{}^{\underline{\alpha}}(x) \;=\; \partial_m z^{\underline{M}} E_{\underline{M}}{}^{\underline{\alpha}} \qquad \text{at } \theta = 0\,. \tag{A.2}$$

are the embedding matrices in the Green-Schwarz formalism. The matrix $\Gamma_{(5)}$ at $\theta = 0$ is given by

$$\Gamma_{(5)} = -\left[\exp\left(-\tfrac{1}{3}\gamma^{mnp}h_{mnp}\right)\right]\Gamma_{(0)}\,, \tag{A.3}$$

where

$$\Gamma_{(0)} = \frac{1}{6!\sqrt{-g}}\epsilon^{m_1\cdots m_6}\gamma_{m_1\cdots m_6}\,. \tag{A.4}$$

The pullback γ-matrices in (A.3) and (A.1) are defined by

$$\gamma_m = \Gamma_{\underline{a}}\mathcal{E}_m{}^{\underline{a}}\,, \qquad \gamma^b = \gamma^m e_m{}^b. \tag{A.5}$$

The matrix $e_m{}^a$ is the vielbein for the induced metric

$$\begin{aligned} g_{mn}(x) &= \mathcal{E}_m{}^{\underline{a}}\mathcal{E}_n{}^{\underline{b}}\eta_{\underline{ab}} \\ &= e_m{}^a e_n{}^b \eta_{ab}\,, \end{aligned} \tag{A.6}$$

and G^{mn} is another metric defined as

$$G^{mn} = (m^2)^{ab} e_a{}^m e_b{}^n\,, \tag{A.7}$$

where

$$m_a{}^b = \delta_a{}^b - 2k_a{}^b, \tag{A.8}$$

$$k_a{}^b = h_{acd}h^{bcd}\,, \tag{A.9}$$

and h_{abc} is a self-dual field strength which is related to H_{abc} via the equation

$$h_{abc} = m_a{}^d H_{cde}\,. \tag{A.10}$$

The G_7 is the seven form that occurs in the dual formulation of eleven dimensional supergravity,

$$G_{\underline{d}_1\cdots\underline{d}_4} = \frac{1}{7!}\epsilon_{\underline{d}_1\cdots\underline{d}_4\underline{e}_1\cdots\underline{e}_7}G^{\underline{e}_1\cdots\underline{e}_7}. \tag{A.11}$$

The target space indices on G_4 and G_7 have been converted to worldvolume indices with factors of $\mathcal{E}_m{}^{\underline{a}}$.

The κ-symmetry transformation rules are

$$\begin{aligned} \delta_\kappa z^{\underline{a}} &= 0\,, \\ \delta_\kappa z^{\underline{\alpha}} &= \kappa^{\underline{\gamma}}(1+\Gamma_{(5)})_{\underline{\gamma}}{}^{\underline{\alpha}}\,, \\ \delta_\kappa h_{abc} &= -\tfrac{i}{16}m_{[a|}{}^d\,\mathcal{E}_d(1-\Gamma_{(5)})\gamma_{|bc]}\kappa\,, \end{aligned} \tag{A.12}$$

where $\Gamma_{(5)}$ is given by (A.3).

JHEP12(1997)001

References

[1] A. Strominger, *Open p-branes*, *Phys. Lett.* **B 383** (1996) 44 [hep-th/9512059].

[2] P.K. Townsend, *D-branes from M-branes*, *Phys. Lett.* **B 373** (1996) 68 [hep-th/9512062].

[3] K. Becker and M. Becker, *Boundaries in M-theory*, *Nucl. Phys.* **B 472** (1996) 221 [hep-th/9602071].

[4] M. Cederwall, *Boundaries of 11-dimensional membranes*, *Mod. Phys. Lett.* **A 12** (1997) 2641-2645 [hep-th/9704161].

[5] Ph. Brax and J. Mourad, *Open membranes in eleven dimensions*, *Phys. Lett.* **B 408** (1997) 142-150 [hep-th/9704165].

[6] K. Ezawa, Y. Matsuo and K. Murakami, *Matrix regularization of open supermembrane - towards M-theory five-brane via open supermembrane*, hep-th/9707200.

[7] Ph. Brax and J. Mourad, *Open supermembrane coupled to M- theory five-branes*, hep-th/9707246.

[8] P.S. Howe and E. Sezgin, *Superbranes*, *Phys. Lett.* **B 390** (1997) 133 [hep-th/9607227].

[9] P.S. Howe and E. Sezgin, *D=11, p=5*, *Phys. Lett.* **B 394** (1997) 62 [hep-th/9611008].

[10] P.S. Howe, E. Sezgin and P.C. West, *Covariant field equations of the M-theory five-brane*, *Phys. Lett.* **B 399** (1997) 49 [hep-th/9702008].

[11] P.S. Howe, E. Sezgin, *Super M-Branes*, in preparation.

[12] P.S. Howe, O. Raetzel, E. Sezgin and P. Sundell, *D-Brane superembeddings*, in preparation.

[13] E. Bergshoeff, E. Sezgin and P.K. Townsend, *Supermembranes and eleven dimensional Supergravity*, *Phys. Lett.* **B 189** (1987) 75.

[14] E. Bergshoeff, E. Sezgin and P.K. Townsend, *Properties of the eleven dimensional supermembrane theory*, *Ann. Phys. (NY)* **185** (1988) 330.

[15] E. Cremmer and S. Ferrara, *Formulation of 11-dimensional supergravity in superspace*, *Phys. Lett.* **B 91** (1980) 61.

[16] L. Brink and P.S. Howe, *Eleven dimensional supergravity on the mass shell in superspace*, *Phys. Lett.* **B 91** (1980) 384.

JHEP12(1997)001

[17] I. Bandos, K. Lechner, A. Nurmagambetov, P. Pasti, D. Sorokin and M. Tonin, *Covariant action for the super-fivebrane of M-theory*, *Phys. Rev. Lett.* **78** (1997) 4332 [hep-th/9701149].

[18] M. Aganagic, J. Park, C. Popescu, and J.H. Schwarz, *Worldvolume action of the M theory fivebrane*, *Nucl. Phys.* **B 496** (1997) 191 [hep-th/9701166].

[19] I. Bandos, K. Lechner, A. Nurmagambetov, P. Pasti, D. Sorokin and M. Tonin, *On the equivalence of different formulations of the M theory fivebrane*, *Phys. Lett.* **B 408** (1997) 135 [hep-th/9703127].

[20] E. Witten, *Five-brane effective action in M-theory*, hep-th/9610234.

[21] S.P. de Alwis, *Coupling of branes and normalizations of effective actions in string/M-theory*, *Phys. Rev.* **D 56** (1997) 7963-7977 [hep-th/9705139].

[22] L. Bonora, C.S. Chu and M. Rinaldi, *Perturbative anomalies of the M-5-brane*, hep-th/9710063.

[23] P. Horava and E. Witten, *Heterotic and type I string dynamics from eleven dimensions*, *Nucl. Phys.* **B 460** (1996) 506 [hep-th/9510209].

[24] C.S. Chu, P.S. Howe and E. Sezgin, in preparation.

[25] B. de Wit, J. Hoppe and H. Nicolai, *On the quantum mechanics of supermembranes*, *Nucl. Phys.* **B 305** (1988) 545.

[26] B. de Wit, U. Marquard and H. Nicolai, *Area preserving diffeomorphisms and supermembrane Lorentz invariance*, *Comm. Math. Phys.* **128** (1990) 39.

[27] N. Kim and S.J. Rey, *M(atrix) theory on an orbifold and twisted membrane*, *Nucl. Phys.* **B 504** (1997) 189-213 [hep-th/9701139].

[28] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, *M theory as a matrix model: a conjecture*, *Phys. Rev.* **D 55** (1997) 5112 [hep-th/9610043].

[29] M.R. Douglas, *D-branes and matrix theory in curved space*, lectures at Strings '97, hep-th/9707228.

[30] M. Berkooz and M.R. Douglas, *Five-branes in M(atrix) theory* , *Phys. Lett.* **B 395** (1997) 196 [hep-th/9610236].

JHEP12(1997)001